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A new product estimator for estimating finite population mean

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Abstract

Using the estimated coefficient of variation of the study variable, a product estimator is proposed which is found to be less biased than the conventional product estimator. Further, it is shown to be more efficient than the exponential product estimator under conditions that hold good in practice. Empirical studies have been carried out to support theoretical investigations.

Keywords: New product estimator, study variable, estimated coefficient of variation, percent relative bias.

1. Introduction

In order to have a survey estimate of $\overline{Y}$, Murthy (1964) proposed the classical product estimator as:

$$\bar{y}_p = \frac{\bar{y}\,\bar{x}}{\bar{X}} \tag{1.1}$$

where the symbols have their usual meanings and $\overline{X}$, the population mean of the auxiliary variable, is assumed to be known in advance. Then, the Bias and the Mean Squared Error (M.S.E.) of $\bar{y}_p$, to the first degree of approximation, are given by:

$$\begin{aligned}\text{Bias }(\bar{y}_p) &= \bar{Y}\,\theta_1\,\rho C_y C_x \\ &= \theta_1 \bar{Y}\, C_{yx} \\ &= \theta_1 \bar{Y}\, C_{11}\end{aligned} \tag{1.2}$$

$$\begin{aligned}\text{MSE}(\bar{y}_p) &= \bar{Y}^2\,\theta_1\left[C_y^2 + C_x^2 + 2C_{yx}\right] \\ &= \theta_1 \bar{Y}^2\left[C_{02} + C_{20} + 2C_{11}\right],\end{aligned} \tag{1.3}$$

where $\theta_1 = \dfrac{N-n}{Nn}$, $C_{02} = C_y^2$, $C_{20} = C_x^2$ and $C_{11} = C_{yx} = \rho C_y C_x$

Srivastava (1983), following the predictive approach due to Basu (1971), suggested the product estimator

$$\bar{y}'_p = \frac{n\bar{y}}{N} + \frac{(N-n)^2\,\bar{y}\,\bar{x}}{N(N\bar{X} - n\bar{x})}, \tag{1.4}$$

and the predictive product estimator due to Agrawal and Jain (1989) is formulated as

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$$\bar{y}_p'' = \frac{\bar{y}\,\tilde{x}}{\tilde{X}}, \tag{1.5}$$

where $\tilde{x}$ and $\tilde{X}$ are, respectively, the sample and population harmonic means of x variable.
The product estimator, which Bahl and Tuteja (1991) proposed, is given by

$$\bar{y}_{ep} = \bar{y}\exp\left[\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right]. \tag{1.6}$$

The expressions for bias and MSE of $\bar{y}_{ep}$, to o(n^{-1}), are given by:

$$B(\bar{y}_{ep}) = \theta_1\bar{Y}\left[\frac{1}{2}C_{11} - \frac{1}{8}C_{20}\right], \tag{1.7}$$

and

$$\text{MSE}(\bar{y}_{ep}) = \theta_1\bar{Y}^2\left[C_{02} + \frac{1}{4}C_{20} + C_{11}\right]. \tag{1.8}$$

2. Proposed Estimator

Incorporating the ideas due to Murthy (1964) and Panigrahi and Mishra (2017), we have developed a new product estimator using Simple Random Sampling without Replacement (SRSWOR). The proposed estimator is compared with the conventional product estimator and the conventional exponential product estimator, both theoretically and empirically.

The new product type estimator is given by

$$\bar{y}_{p_1} = \bar{y}\left(1+\theta_1\hat{C}_y^2\right)\frac{\bar{x}}{\bar{X}}, \tag{2.1}$$

where $\hat{C}_y^2 = \frac{\hat{S}_y^2}{\bar{y}^2}$ is the estimate of C_y^2, $\frac{\hat{S}_y^2}{\bar{y}^2} = \frac{s_y^2}{\bar{y}^2}$

and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2$.

The Bias and Mean Squared Error (M.S.E.) of the proposed estimator $\bar{y}_{p_1}$ are obtained as:

$$B(\bar{y}_{p_1}) = E(\bar{y}_{p_1} - \bar{Y}) = \theta_1\bar{Y}\left[C_{11} + C_{02}\right] \tag{2.2}$$

$$MSE(\bar{y}_{p_1}) = E(\bar{y}_{p_1} - \bar{Y})^2 = \theta_1\bar{Y}^2\left[C_{02} + C_{20} + 2C_{11}\right]. \tag{2.3}$$

3. Comparison of Bias

It is observed that, if we consider the first order of approximation, the conventional product estimator and the proposed product estimator are coming out to be equally efficient. Therefore, we compare the percent relative bias of the conventional product estimator with that of the proposed product estimator and the conventional exponential product estimator.

The product estimator $\bar{y}_{p_1}$ is less biased than the conventional product estimator $\bar{y}_p$ if

$$2C_{11} + C_{02} \le 0$$

or

$$C_{02} \le -2C_{11} \qquad (3.1)$$

is satisfied.

4. Comparison of Efficiency

Here, we will consider the MSEs of the proposed product estimator and the exponential product estimator.

The proposed product estimator will be more efficient than the exponential product estimator if

$$4C_{11} + 3C_{20} \le 0$$

or

$$C_{20} \le -\frac{4}{3}C_{11} \qquad (4.1)$$

is satisfied.

5. Empirical Investigation

As we know that the classical product estimator and the proposed product estimator are equally efficient, so here we will study the percent relative bias between these estimators. The biases are calculated from the given population. Table 5.1 gives the population chacteristics with correlation coefficient (ρ) and the coefficient of variation C_x and C_y . Table 5.2 gives the absolute bias of the estimators $\bar{y}_p, \bar{y}_{p_1}$ and $\bar{y}_{ep}$. Table 5.3 and Table 5.4 give the percent relative bias of the estimators $\bar{y}_p, \bar{y}_{p_1}$ and $\bar{y}_{ep}$ for sample sizes 4 & 5 respectively.

For efficiency comparison of proposed product estimator $\bar{y}_{p_1}$ and the exponential conventional product estimator $\bar{y}_{ep}$, Table 5.5 gives the population chacteristics with correlation coefficient (ρ) and the coefficient of variations C_x , C_y and Table 5.6 gives the MSE of the estimators $\bar{y}$, $\bar{y}_{ep}$ and $\bar{y}_{p_1}$.

Table: 5.1
Population Characteristics

Population no.	Source	X	Y	N	ρ	C_x	C_y
1	Black (2009) P.476	Number of Farms (in millions)	Average Size (in acres)	12	-0.98	0.384	0.223
2	Black (2009) P. 553	Price of Stock	Price of Stock	15	-0.68	0.339	0.149
3	Black (2009) P. 553	Price of Stock	Price of Stock	15	-0.67	0.349	0.149

4.	Draper and Smith (1966) P. 352	Average Atmospheric Temperature	Amount of Steam Used per Month	25	-0.84	0.328	0.173
5	Draper and Smith (1966) P. 352	Average Atmospheric Temperature	Average Wind Velocity	25	-0.61	0.328	0.275
6	Hardle and Hlavka (2007) P.335	Marks for Car Safety	Marks for Car Price	24	-0.7	0.334	0.360
7	Hardle and Hlavka (2007) P.339	Expenditure of Fruits	Expenditure on Wine	12	-0.48	0.326	0.194
8	Kenney and Keeping (1966) P.310	Age of Males	Mean Vital Capacity	18	-0.93	0.355	0.117
9	Draper and Smith (1966) P. 366	Weight Percent of Dicalcium Silicate	Weight Percent of Tricalcium Silicate	13	-0.97	0.557	0.323
10	Maddala (1988) P.109	Unemployment Rate	Quit Rate per Hundred Employees	13	-0.81	0.291	0.273

Table: 5.2
Absolute bias of the estimators
(Natural and Artificial Populations)

Population No.	$\bar{y}_{ep}$	$\bar{y}_{p}$	$\bar{y}_{p_1}$
1	0.06	0.084	0.034
2	0.031	0.034	0.012
3	0.033	0.035	0.013
4	0.038	0.048	0.018
5	0.042	0.055	0.021
6	0.056	0.084	0.046
7	0.028	0.030	0.008
8	0.036	0.039	0.025
9	0.127	0.175	0.071
10	0.043	0.064	0.011

Table: 5.3
The percent relative bias of the estimators
(Natural and Artificial Populations)
Sample size n=4

Population No.	$\bar{y}_{ep}$	$\bar{y}_{p}$	$\bar{y}_{p_1}$
1	44.622	20.205	8.18
2	10.186	5.521	1.969
3	10.529	5.521	2.054
4	18.32	10.717	4.030
5	8.885	9.389	0.036
6	9.394	14.090	7.706
7	6.135	4.213	1.145
8	18.876	6.908	4.4
9	62.899	28.787	11.69
10	9.984	14.893	2.538

Table: 5.4
The percent relative bias of the estimator
(Natural and Artificial Populations)
Sample size n=5

Population No.	$\bar{y}_{ep}$	$\bar{y}_{p}$	$\bar{y}_{p_1}$
1	37.312	16.895	6.84
2	8.687	4.709	1.679
3	8.979	4.709	1.752
4	16	9.36	3.52
5	7.76	8.2	3.12
6	8.178	12.267	6.709
7	5.13	3.523	0.958
8	16.259	5.950	3.79
9	53.071	24.289	9.863

10	8.424	12.566	2.141

Table: 5.5
Population Characteristics

Population	Source	X	Y	N	ρ	C_x	C_y
1	Black (2009) P.476	Year	Number of Farms (in millions)	12	-0.81	0.009	0.384
2	Black (2009) P. 521	Year	Personal Savings	15	-0.96	0.004	0.618

Table: 5.6
MSE of the estimators
(Natural and Artificial Populations)

Population No.	$\bar{y}$	$\bar{y}_{ep}$	$\bar{y}_{p_1}$
1	0.147	0.144	0.141
2	0.382	0.380	0.377

6. Conclusion

The empirical findings presented in Tables 5.2, 5.3, 5.4 and 5.6 clearly demonstrate the superior performance of the proposed estimator $\bar{y}_{p_1}$ in terms of both bias and efficiency over the conventional alternatives $\bar{y}_p$ and $\bar{y}_{ep}$ and for all the populations shown in Table 5.2**,** the absolute bias of the proposed estimator $\bar{y}_{p_1}$ is consistently lower than that of the corresponding competing estimators $\bar{y}_p$ and $\bar{y}_{ep}$. Similarly, the findings reported in Tables 5.3 and 5.4 reveal that the percent relative bias of the proposed estimator $\bar{y}_{p_1}$ remains smaller than that of both the conventional product estimator $\bar{y}_p$ and the conventional exponential product estimator $\bar{y}_{ep}$, confirming its improved accuracy. Moreover, for both the Populations 1 and 2 in Table 5.6**,** the Mean Squared Error (MSE) associated with the proposed estimator $\bar{y}_{p_1}$ is found to be less than that of the competing estimator $\bar{y}_{ep}$ indicating its greater precision and stability. Overall, as the proposed estimator $\bar{y}_{p_1}$ exhibits both lower bias and smaller MSE almost all populations considered, it may be regarded as a more efficient and reliable alternative to the existing estimators of the population mean.

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